

On bouncing solutions in non-local gravity

Alexey S. Koshelev^{1*} and Sergey Yu. Vernov^{2,3†}

¹Theoretische Natuurkunde, Vrije Universiteit Brussel and The International Solvay Institutes,
Pleinlaan 2, B-1050 Brussels, Belgium

²Instituto de Ciencias del Espacio (ICE/CSIC) and
Institut d'Estudis Espacials de Catalunya (IEEC),

Camp. UAB, Fac. Ciències, T. C5, E-08193, Bellaterra, Barcelona, Spain

³Skobeltsyn Institute of Nuclear Physics, Lomonosov Moscow State University,
Leninskie Gory 1, 119991, Moscow, Russia

Abstract

A non-local modified gravity model with an analytic function of the d'Alembert operator is considered. This model has been recently proposed as a possible way of resolving the singularities problem in cosmology. We present an exact bouncing solution, which is simpler compared to the already known one in this model in the sense it does not require an additional matter to satisfy all the gravitational equations.

1 Introduction

Modified gravity cosmological models have been proposed with the hope to find resolutions to the important open problems of the standard cosmological model. One possible modification which allows to improve the ultraviolet behavior and even to get a renormalizable theory of quantum gravity is the addition of higher-derivative terms to the Einstein–Hilbert action (as one of the first papers we can mention [1]). Unfortunately, models with the higher-derivative terms have ghosts. A possible way to overcome this problem is to consider a non-local gravity.

The main theoretical motivation for studying cosmological models with the non-local corrections to the Einstein–Hilbert action comes from the string field theory [2]. These corrections usually contain exponential functions of the d'Alembertian operator and appear in such stringy models as effective tachyonic actions. The majority of the non-local cosmological models motivated by such structures explicitly include an analytic or meromorphic function of the d'Alembert operator [3, 4, 5, 6, 7, 8, 9].

Usually both the general relativity and the modified gravity models are described by a nonintegrable system of equations and only particular exact solutions can be obtained. At the same time exact solutions play an important role in the cosmological models since one must consider perturbations in order to claim the model is realistic. Needless to say exact solutions for non-local nonlinear equations is an extremely tough subject. Some studies for nonlocal gravitational models with exact solutions can be found in [5, 6, 7, 8].

*Postdoctoral researcher of FWO-Vlaanderen, E-mail: alexey.koshelev@vub.ac.be

†E-mail: vernov@ieec.uab.es, svernov@theory.sinp.msu.ru

2 Action and Equations of Motion

The nonlocal modification of the Einstein gravity, which has been proposed in [5, 6], is described by the following action:

$$S = \int d^4x \sqrt{-g} \left(\frac{M_P^2}{2} R + \frac{1}{2} R \mathcal{F}(\square/M_*^2) R - \Lambda \right), \quad (1)$$

where M_P is the Planck mass. M_* is the mass scale at which the higher derivative terms in the action become important. An analytic function $\mathcal{F}(\square/M_*^2) = \sum_{n \geq 0} f_n \square^n$ is an ingredient inspired by the SFT. The operator \square is the covariant d'Alembertian. In the case of an infinite series we have a non-local action.

Let us introduce dimensionless coordinates $\bar{x}_\mu = M_* x_\mu$ and $\bar{M}_P = M_P/M_*$. It is easy to see that $\mathcal{F}(\square/M_*^2) = \mathcal{F}(\bar{\square})$, where $\bar{\square}$ is the d'Alembertian in terms of dimensionless coordinates. In the following we omit bars using only dimensionless coordinates.

A straightforward variation of action (1) yields the following equations

$$\begin{aligned} (M_P^2 + 2\mathcal{F}(\square)R) \left(R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} \right) &= 2(D_\mu \partial_\nu - g_{\mu\nu} \square) \mathcal{F}(\square) R - \Lambda g_{\mu\nu} + \\ &+ \frac{1}{2} \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left[\partial_\mu \square^l R \partial_\nu \square^{n-l-1} R + \partial_\nu \square^l R \partial_\mu \square^{n-l-1} R - \right. \\ &\left. - g_{\mu\nu} (g^{\rho\sigma} \partial_\rho \square^l R \partial_\sigma \square^{n-l-1} R + \square^l R \square^{n-l} R) \right] - \frac{1}{2} R \mathcal{F}(\square) R g_{\mu\nu}, \end{aligned} \quad (2)$$

where D_μ is the covariant derivative. It is useful (see [6]) to write down the trace equation:

$$M_P^2 R - \sum_{n=1}^{\infty} f_n \sum_{l=0}^{n-1} \left(\partial_\mu \square^l R \partial^\mu \square^{n-l-1} R + 2 \square^l R \square^{n-l} R \right) - 6 \square \mathcal{F}(\square) R = 4\Lambda. \quad (3)$$

3 General Ansatz for finding Exact Solutions

It has been shown in [5] that the following ansatz

$$\square R = r_1 R + r_2, \quad (4)$$

with $r_1 \neq 0$, is useful in finding exact solutions. Using (4), the trace equation becomes

$$A_1 R + A_2 (2r_1 R^2 + \partial_\mu R \partial^\mu R) + A_3 = 0 \quad (5)$$

where

$$A_1 = -M_P^2 + 4\mathcal{F}'(r_1)r_2 - 2\frac{r_2}{r_1}(\mathcal{F}(r_1) - f_0) + 6\mathcal{F}(r_1)r_1, \quad A_2 = \mathcal{F}'(r_1), \quad A_3 = 4\Lambda + \frac{r_2}{r_1}M_P^2 + \frac{r_2}{r_1}A_1.$$

The simplest way to get a solution to equation (5) is to put all the above coefficients to zero. Relations $A_j = 0$, $j = 1, 2, 3$ determine values of r_1 , r_2 and also fix the cosmological constant:

$$\mathcal{F}'(r_1) = 0, \quad r_2 = -\frac{r_1[M_P^2 - 6\mathcal{F}(r_1)r_1]}{2[\mathcal{F}(r_1) - f_0]}, \quad \Lambda = -\frac{r_2 M_P^2}{4r_1} = M_P^2 \frac{[M_P^2 - 6\mathcal{F}(r_1)r_1]}{8[\mathcal{F}(r_1) - f_0]}. \quad (6)$$

4 Exact solutions and their applications

Let us consider solutions in the spatially flat Friedmann–Lemaître–Robertson–Walker (FLRW) metric with the interval $ds^2 = -dt^2 + a^2(t)(dx_1^2 + dx_2^2 + dx_3^2)$.

The very important result for this kind of models was a construction of an analytic solution describing the non-singular bounce

$$a(t) = a_0 \cosh(\lambda t), \quad (7)$$

where a_0 is an arbitrary constant and $\lambda = \sqrt{\Lambda}/3M_P$. To satisfy all equations (2) one should add some radiation to the model. This is the exact analytic result and we refer the readers to [5, 6] about all the details.

Let us consider another solution, which satisfies the ansatz (4). Namely,

$$a(t) = a_0 e^{\frac{\lambda}{2}t^2}, \quad (8)$$

where a_0 is an arbitrary constant. On this solution

$$H(t) = \lambda t, \quad R = 12\lambda^2 t^2 + 6\lambda, \quad \square R = -72\lambda^3 t^2 - 24\lambda^2, \quad \Rightarrow \quad r_1 = -6\lambda, \quad r_2 = 12\lambda^2, \quad (9)$$

where $H = \dot{a}/a$ is the Hubble parameter and the dot denotes the differentiation with respect to time t . From the condition $A_3 = 0$ we get $\Lambda = \lambda M_P^2/2$. From $A_1 = 0$ and $A_2 = 0$ we get the following constraints on the function \mathcal{F} and its first derivative in the point r_1 :

$$\mathcal{F}(r_1) = -\frac{M_P^2}{32\lambda} - \frac{f_0}{8}, \quad \mathcal{F}'(r_1) = 0. \quad (10)$$

There are two independent Einstein equations in the FLRW metric. Let us consider "00" component of system (2), which, after imposing the simplifying ansatz and using conditions $A_j = 0$, reads as the second order differential equation for the Hubble parameter $H(t)$:

$$\mathcal{F}(r_1) \left[H\ddot{H} + 3H^2\dot{H} - \frac{1}{2}\dot{H}^2 + \frac{r_1}{2}H^2 + \frac{r_2}{24} \right] = 0. \quad (11)$$

Substituting (9), we obtain that equation (11) is satisfied, so function (8) is a solution to all Einstein equations. Note that we do not add any matter to get the exact solution.

We stress that a construction of exact solutions is obviously a non-trivial task and to the moment only one non-trivial exact analytic bouncing solution (7) in the class of models given by action (1) is analyzed [6]. We present here another bouncing solution (8) which is simpler compared to (7) in the sense it does not require an additional matter to be present.

We are leaving open the questions of perturbation spectrum for exact solutions in this nonlocal model and those applications to describing the bounce phase and the initial inflation stage. These question will be addressed in the forthcoming publications.

Acknowledgments. The authors are grateful to the organizers of the Dubna International SQS'11 Workshop for the hospitality and the financial support. The authors thank T. Biswas for very useful and stimulating discussions. The work is supported in part by the RFBR grant 11-01-00894. A.K. is supported in part by the Belgian Federal Science Policy Office through the Interuniversity Attraction Pole P6/11, and in part by the "FWO-Vlaanderen" through the project G.0114.10N. S.V. is supported in part by grant of the Russian Ministry of Education and Science NSh-4142.2010.2 and by contract CPAN10-PD12 (ICE, Barcelona, Spain).

References

- [1] Stelle K.S., Renormalization Of Higher Derivative Quantum Gravity, Phys. Rev. D 1977. V. 16. P. 953-969
- [2] Aref'eva I.Ya., Belov D.M., Giryavets A.A., Koshelev A.S., Medvedev P.B., Noncommutative Field Theories and (Super)String Field Theories, hep-th/0111208
- [3] S. Jhingan, S. Nojiri, S.D. Odintsov, M. Sami, I. Thongkool, S. Zerbini, Phantom and non-phantom dark energy: The cosmological relevance of non-locally corrected gravity, Phys. Lett. B 2008. V 663. P. 424–428, arXiv:0803.2613;
Koivisto T.S., Dynamics of Nonlocal Cosmology, Phys. Rev. D **77** (2008) 123513, arXiv:0803.3399;
Capozziello S., Elizalde E., Nojiri S., Odintsov S.D., Accelerating cosmologies from non-local higher-derivative gravity, Phys. Lett. B 2009, V. 671. P. 193–198, arXiv:0809.1535;
Calcagni G., Nardelli G., Nonlocal gravity and the diffusion equation, Phys. Rev. D 2010. V. 82. P. 123518, arXiv:1004.5144;
Leonardo Modesto, Super-renormalizable Quantum Gravity, arXiv:1107.2403.
- [4] Barnaby N., Biswas T., Cline J.M., p-adic Inflation , JHEP 2007. V. 0704. P. 056, hep-th/0612230;
Koshelev A.S., Non-local SFT Tachyon and Cosmology , JHEP 2007. V. 0704. P. 029, hep-th/0701103;
Aref'eva I.Ya., Joukovskaya L.V., Vernov S.Yu., Bouncing and accelerating solutions in nonlocal stringy models JHEP 2007. V. 0707. P. 087, hep-th/0701184;
Mulryne D.J., Nunes N.J., Diffusing non-local inflation: Solving the field equations as an initial value problem , Phys. Rev. D 2008 V. 78. P. 063519, arXiv:0805.0449;
Aref'eva I.Ya., Koshelev A.S., Cosmological Signature of Tachyon Condensation , JHEP 2008. V. 0809. P. 068., arXiv:0804.3570;
Koshelev A.S., Vernov S.Yu., Analysis of scalar perturbations in cosmological models with a non-local scalar field , Class. Quant. Grav. 2011 V. 28. P. 085019, arXiv:1009.0746;
Alexey S. Koshelev, Modified non-local gravity, arXiv:1112.6410.
- [5] Biswas T., Mazumdar A., Siegel W., Bouncing Universes in String-inspired Gravity , JCAP 2006 V. 0603. P. 009, hep-th/0508194
- [6] Biswas T., Koivisto T., Mazumdar A., Towards a resolution of the cosmological singularity in non-local higher derivative theories of gravity , JCAP 2010. V. 1011. P. 008, arXiv:1005.0590
- [7] Aref'eva I.Ya., Joukovskaya L.V., Vernov S.Yu., Dynamics in nonlocal linear models in the Friedmann–Robertson–Walker metric , J. Phys. A: Math. Theor. **41** (2008) 304003, arXiv:0711.1364;
Vernov S.Yu., Localization of the SFT inspired Nonlocal Linear Models and Exact Solutions , Phys. Part. Nucl. Lett. 2011. V. 8. P. 310–320, arXiv:1005.0372
- [8] Nojiri Sh., Odintsov S.D., Modified non-local-F(R) gravity as the key for the inflation and dark energy, Phys. Lett. B 2008. V. 659. P. 821, arXiv:0708.0924;

- Zhang Y.l., Sasaki M., Screening of cosmological constant in non-local cosmology, Int. J. Mod. Phys. D 2012. V. 21. P. 1250006, arXiv:1108.2112;
- Elizalde E., Pozdeeva E.O., Vernov S.Yu., De Sitter Universe in Non-local Gravity, Phys. Rev. D 2012. V. 85. P. 044002, arXiv:1110.5806
- [9] T. Biswas, E. Gerwick, T. Koivisto, A. Mazumdar, Towards singularity and ghost free theories of gravity, Phys. Rev. Lett. 2012 V. 108 P. 031101, arXiv:1110.5249